1.1 Simple Interest

Friday, February 01, 2013
11:12 AM

Ex If you invest $100 in a simple interest bank acct on your b'day, that earns 5% (per year), how much will you have on your next birthday?

$$0.05 \times \$100 = \$5$$

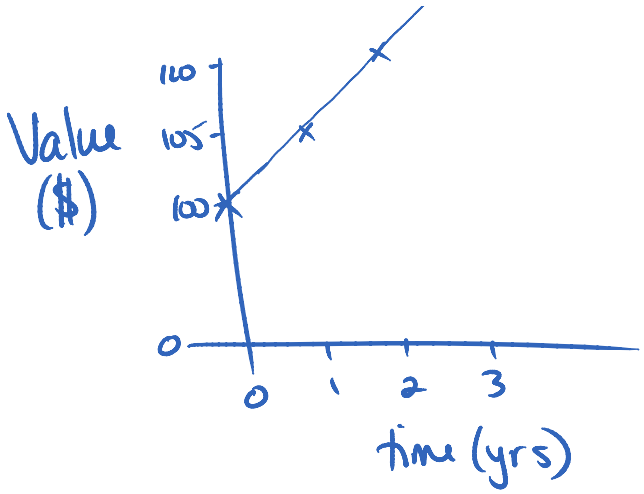
$$\$100 + \$5 = \$105$$

- this was a fixed interest rate (guarenteed not to change)
- the $100 was the principal (initial investment)
- one year was the term (agreed upon length of the investment)
- the investment matured (ended) on your birthday

Ex Now assume that you let the investment remain invested for 10 years (at 5%), instead of 1 yr. How much interest would you earn?

year	interest	total
		100

year	interest	total
0	—	100
1	$5	105
2	$5	110
3	$5	115
⋮	⋮	⋮



$5 per year × 10 years = $50

– this is simple interest: you only get interest on the original principal ($100); there is no interest on the interest (that would be compound – next section)

Formula

$$A = P + Prt$$

A: future value; P: principal; r: interest rate (decimal form); t: # of years

$$A = P(1+rt)$$

$$I = Prt$$

I = interest earned

$\text{rate of return} = \frac{I}{P}$

1.2 & 1.3 Compound Interest Notes

Monday, February 04, 2013
10:13 AM

Compound Interest – interest that is paid on the principal plus on any accumulated interest (interest on the interest)

Ex $1000 @ 3.6% for 5 years

simple interest $I = Prt$

$= 1000(0.036)(5)$

$= \$180$ ($\to \$36$ per year)

$\to$ \$1180 Total

compound interest (compounding annually)

end of 1st year: $1000 + \underset{\text{interest}}{\$36} = \$1036$

end of 2nd yr: $1036 + 37.30 = \$1073.30$

$I = 1036(0.036)$

end of 3rd yr: $1073.30 + 38.64 = 1111.94$

$I = 1073.30(0.036)$

end of 4th year: $1111.94 + 40.03 = 1151.97$

end of 5th year: $1151.97 + 41.47 = \$1193.44$ Total

∴ Compound interest earns you more money.

Formula

A simpler way to calculate compound interest

$$A = P\left(1 + \frac{i}{n}\right)^{tn}$$

- A: Future value
- P: principal
- i: Note diff. than formula in text
- n: # of compounding periods in a year
- t: # of years

compounding annually: $n=1$
semi-annual: $n=2$
monthly: $n=12$
quarterly: $n=4$

for:

Future Value | principal | interest rate (annual) | # of compounding periods in a year

semi-annual: n = 2
monthly: n = 12
quarterly: n = 4
weekly: n = 52
daily: n = 365

back to example

$P = 1000$
$i = 0.036$
$n = 1$
$t = 5$
$A = ?$

$$A = 1000\left(1 + \frac{0.036}{1}\right)^{5(1)}$$
$$= 1000(1.036)^5$$
$$= \$1193.44$$

slight change — compounding monthly

$$A = 1000\left(1 + \frac{0.036}{12}\right)^{5(12)}$$
$$= 1000(1.003)^{60}$$
$$= \$1196.89$$

Interest Earned $= A - P$

$= 1196.89 - 1000 = \$196.89$

Rule of 72

- estimates the doubling time of an investment
- works best with compound interest that compounds annually.

Formula $\frac{72}{r}$

r ← annual rate of interest as a %

r ← annual rate of interest as a %

back to ex

$$\frac{72}{3.6\%} = 20 \text{ years to double}$$

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pg 30 # 4a,c, 5, 7, 9, 13

1.4 Finding Present Value (Compound Interest)

Tuesday, February 05, 2013
10:38 AM

$$A = P\left(1 + \frac{i}{n}\right)^{tn}$$

$$P = \frac{A}{\left(1 + \frac{i}{n}\right)^{tn}}$$

Finding interest rate:

$$\sqrt[tn]{\frac{A}{P}} = \sqrt[tn]{\left(1 + \frac{i}{n}\right)^{tn}}$$

$$\sqrt[tn]{\frac{A}{P}} = 1 + \frac{i}{n}$$

$$\sqrt[tn]{\frac{A}{P}} - 1 = \frac{i}{n}$$

$$n\left(\sqrt[tn]{\frac{A}{P}} - 1\right) = i$$

Finding t:

$$A = P\left(1 + \frac{i}{n}\right)^{nt}$$

given A, P, i and n

$$\frac{A}{P} = \left(1 + \frac{i}{n}\right)^{nt}$$

guess the "t"

Comparing Investments:

look at the ratio $\frac{A}{P} = \left(1 + \frac{i}{n}\right)^{nt}$

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① Prediction → B - quarterly - because it is compounded less frequently, so requires more to start with in order to end up at the same place in

10 years.

(2a) option A: $\frac{A}{P} = \left(1 + \frac{0.05}{12}\right)^{10(12)} = 1.647$ ← increasing faster

(12 ↑ monthly)

option B: $\frac{A}{P} = \left(1 + \frac{0.05}{4}\right)^{10(4)} = 1.644$

(4 ↑ quarterly)

b) $\frac{A}{P} = \left(1 + \frac{0.06}{1}\right)^{10(1)} = 1.791$ → Effectively 79% over 10 years or 7.9% per year instead of 6% per year (interest on interest)

(1 ↑ annually)

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